THE PARTHENON

ITS SCIENCE OF FORMS

one of 225 copies.

THE PARTHENON
ITS SCIENCE OF FORMS

The Parthenon

ITS SCIENCE OF FORMS

BY

ROBERT W. GARDNER

THE NEW YORK UNIVERSITY PRESS

32 WAVERLY PLACE, NEW YORK CITY

MCMXXV

THE NEW YORK UNIVERSITY PRESS
ARTHUR HUNTINGTON NASON, PH.D., *Director*

TABLE OF CONTENTS

The Parthenon: Its Science of Forms

To discuss a subject whereon so many "last words" have been written and spoken, and to ask for a hearing on a new aspect of that subject when other aspects have been finally laid to rest, is indeed difficult. If, however, only a lively discussion of this topic is engendered by the presentation of the accompanying set of drawings, the labor involved in their preparation will not have been in vain.

The question, discussed intermittently since in 1787 the publications of Stuart and Revett made known the works of the Greeks to modern times, is whether or not a geometrical law is manifest in the works of art of the Greeks before and at the time of the climax of the Athenian civilization, a climax marked by the design and erection of the buildings on the Acropolis of Athens; whether or not, in addition to an innate feeling for beauty and art, the Greeks possessed a scientific basis for this work, a definite law of order, a canon, not of design, but of proportion.

If there be such a law or canon, we should expect it to pervade the whole Athenian art: its poetry, its music, its vases, its temples, its paintings, its sculpture, and its military and naval architecture. Such a law would bind these separate manifestations into a unity. It would be the fundamental principle that would bring and compel conformity to the Hellenic ideal.

This Hellenic ideal—into the nature of which we need not here inquire—is expressed in one of its highest forms in the architecture and sculpture of the temple at Athens known as the Parthenon. It is this specific work that will be here discussed, together with its bearings on the design of the city and walls of Athens and the naval arsenal which was at its port of Piræus.

Opposed to the conception of a mathematical or geometrical law, or scientific underlying principle of proportion, are those students and writers some of whom, like Statham,[1] hold that, if there be such a law, an attempt to discover or to establish it leads to contradiction and confusion. Anderson and Spiers say, "The highest flights of literature and architecture present an almost perfect parallel. Both have more of art than science, and, down the ages, show little progress within themselves while they clearly reflect the progress of the soul of man."[2] The case for the opposition seems to be summed up very aptly by Barker: "Moreover it would seem that the best opinion is tending back to the idea originally expressed by Hoffer and now ably supported by Goodyear with new facts at his disposal, the idea that to the Greeks design was a matter of feeling. This is far from saying that the feeling was irresponsible or without a guide; on the contrary, the fact that all Greek motives, derived from whatever source, Egyptian, Asiatic, or native, gradually conform to a distinct and special Hellenic ideal, forbids the thought. It would appear then, and the best opinion tends that way, that it is in form and line as such that Greek art is preëminent and that the Greek ideal, whatever its nature, found its expression in form and line."[3] These writers have either left the ultimate question entirely open or have advanced tentative theories; but Perrot and Chipiez, in discussing the geometrical analysis of designs, bring the matter to a conclusion by saying: "Whatever their degree of exactness,

1. Statham, *Short Crit. History of Architecture*, p. 89.
2. Anderson and Spiers, *Architecture of Greece and Rome*, p. 76.
3. Albert W. Barker, *The Subjective Factor in Greek Architectural Design*. A. J. of Archaeology, XX, 1, p. 7.

operations of this kind, which may be executed in indefinite number, are, in fact, more specious than convincing."[4]

On this subject of Geometrical Analysis, which the writer wishes to differentiate from synthesis, or the precise building up of a design by geometrically proportioned forms, there is an article by Dinsmoor in which its author effectually disposes of all the geometrical methods of analysis hitherto published. His criticisms are fair and adequate, and leave nothing to be said so far as the destructive portion of the matter is concerned; but constructively he says, "During an investigation of the problem extending over a dozen years, I was forced in spite of myself to see that the method adopted [in the design of the Parthenon] was hardly different from that of modern times."[5] This seems to lead to the predicament which is virtually confessed when he says that, in so far as their methods of design are concerned, the plan and elevations are unrelated: "While the outer shell was executed exactly as planned, the evolution of the architect's idea literally translated into marble, the case is quite different with the inner building. Here we have an example of the constant revision and alteration to which most designs were subjected in the course of erection." In summing up, he says: "Throughout the whole, however, it is apparent that the designers were working on the basis of centuries of precedent, following definite standards of proportions and detail, standards which varied but slightly in each generation."

From how far back in the past, from what obscure tributaries in older civilizations, this stream of precedent flowed, it would be hard to say. It may have been two thousand years before the Christian era that the direct ancestors of the builders of the Acropolis settled around the eastern shores of the Mediterranean; but it was not until the seventh century B.C. that definite dates appear. These dates refer more to political changes than to matters of art; but the destructive invasion of Greece by the Persian armies of Darius in 490 B.C. and by his son Xerxes in 480 B.C., can probably be said to have ended definitely the formative period of Greek art. With the sea power of the Persians shattered at Salamis, and their army crushed at Platæa, the receding wave of invasion left behind a twice ruined and razed city and Acropolis, but gave the triumphant Athenians an opportunity for the display of genius never equalled in the history of the world.

According to D'Ooge, in the years immediately following the invasion, between 480 and 450, the resources of the state were devoted to the rebuilding of the lower city and of the walls and the fortifications to protect and strengthen them.[6] During this period, also, the town and heavy fortifications around the harbor of the Piræus, five miles from the Acropolis, were planned and built, together with the walled communicating passage connecting the city and the port. All of this was planned, and most of it executed, under the administration of Themistocles. Later, after the ostracism of Themistocles, the work was carried on under Cimon. Still later, after the ruin and banishment of Cimon, the work was taken up by Pericles in the year 461. It was not until about 447 that the national enthusiasm, still at a high pitch after the successful resistance to the Persians, and with the means at its disposal, began to express itself in the rebuilding of the Acropolis. In the incredibly short span of ten years, the work was finished. Thus the entire project was completed in the space of forty-three years.

It is one of the purposes of the present writer to show, by means of the drawings of the Parthenon and of the Acropolis taken from the measurements of Francis Penrose, by the map of the walled city of Athens, restored by Curtius,

4. Perrot and Chipiez, *Histoire d'Art.* Cf. William Bell Dinsmoor, *How the Parthenon Was Planned.* Architecture, Vol. XLVII, Nos. 6 and 7, pp. 180 and 241.

5. Dinsmoor, *ut supra.*

6. Martin L. D'Ooge, *The Acropolis at Athens*, 1908.

and by the map of the town and port of the Piræus with its connecting walls, by Milchöffer, that the whole plan of rebuilding the city, the walls, the temples, the Acropolis, the Parthenon, and the Piræus, was conceived as a unit and so ultimately carried out.

In a word, the unit of measurement of the temple is a geometrical property of, and interchangeable with, the unit of land measurement, the stadion. The system of measurements and the methods of proportioning by square and compasses used by the architects, sculptors, and masterbuilders in the Parthenon, were parts of a larger system of geodesy, which was applied in precisely the same manner to the land measurements and proportions of the city and its environments.

Stated in the language of mathematics, it may be said that all parts of the Parthenon, the Acropolis, the city as rebuilt, the walls, and the port, are expressable in the terms of a geometrically progressing series, which in turn was founded on the standard interchangeable units of Attic foot and Attic stadion.

The names inseparably connected with the Parthenon are those of the sculptor Phidias, the architect Ictinus, the master builder Callicrates, and, commonly, Pericles to the exclusion of Themistocles and Cimon. One cannot mention these last three without associating with them the teacher and philosopher Anaxagoras of Clazomenæ who, in the thirty years of his sojourn in Athens, numbered among his hearers Pericles, Aspasia, Socrates, and Euripides. He taught that a supreme mind, distinct from the visible universe, imparted form and order to the chaos of nature. With this, he taught the geometry and the philosophy of Thales, who, in the sixth century B.C., obtained much of his philosophy and possibly his geometry from Egypt.

Not until the close of the eighteenth century did the modern world awake to the beauties of Hellenic art. We have seen that it still denies that these beauties have a scientific foundation. A half century later, Penrose made his careful and exhaustive study of the Parthenon. His measurements published in *The Principles of Athenian Architecture* are probably the most painstaking and conscientious ever taken of any building. These measurements have been followed by the author in his drawings as closely as possible. The scaled dimensions taken from Penrose and the same quantities obtained geometrically by the immutable law of the series, as later defined, are so nearly identical that it is practically impossible to show in a scale drawing the variations between them. These variances are therefore noted on the margin of the drawings.

The fact is familiar, but must be taken into account, that in Greek art individual members which are apparently "repeats," in actuality vary considerably. This variation occurs especially in the horizontal dimensions at the pavement levels, in the column spacing, in the column diameters, and in the widths of the steps. But, running through these variations and marked by prominence of position, as in the angle columns, or in the centre of the ends as in the metope, or in the plumb-line projections of the angle triglyphs, there can be found a continuous line of related prototypes, true to the geometrical ideal. These descend in unbroken geometric series with no missing links, by geometrical evolution or filiation, carried on not by the foot-rule or measuring stick but by the square and compasses alone, from a single unit at the beginning to the final dimensions of the flanks and roof. From the structure of the Parthenon, the series broadens out to the enclosure of the Acropolis, marking the levels, the steps, the statue of Athena, the locations of the other temples, the salients, angles, and buttresses of its walls, with intermediate steps in the progression indicated by massive marble platforms and slabs on Piræan stone foundations. From the Acropolis, the geometric series extends out through the walled city locating the positions of the

temples, the Stadium, the altar place on the Pnyx, the cross roads, and the wall itself with its gates and buttresses, its salients, towers, and reëntering angles. From the city walls, the long axis of the Parthenon and the diagonal axis of the cella are prolonged as base lines, one ending on the land, near or at the summit of the Mountain of Pausanius, forty stadia from the axial intersections of the cella, the other on the sea, in the sweep of the same arc, forty stadia to the Island of Stalida.

On the diagonal to the Island of Stalida, which appears to be the true base line, the dimensions of the progressing terms may be read alternately in even numbers of Attic feet and of even stadia. Thus, reading the half diagonals to the centre of the Parthenon they would read: forty stadia,—ten thousand feet,—eight stadia, —two thousand feet,—one and six-tenths stadia,—four hundred feet—and so on, each term in succession being a submultiple of the square root of five, or in alternate terms submultiples of five. (See the foot-note in which the series is carried to the ten thousand stadia term.[7])

Thus the stadion and the Attic foot are terms in the same geometrical series of progression; and, by the law of their application, they appear commensurably not on the sides and ends of the squares and rectangles that mark the terms of the series but on the diagonals of the rectangles that relate the successive terms one to the other. This is shown on the map of the Acropolis drawn from Penrose's measurements, Plate II, and the maps of city and port on Plate III.

In any inquiry, the results are related to pure science in the proportion that they may be expressed in the terms of mathematics. To justify, therefore, the title "Science of Forms," the underlying theory of proportion must be of such a nature that it may be expressed mathematically.

The following theorems are here presented, with the brief definitions of the terms used. The demonstrations of these theorems, in the first place, must be apparent in the geometrical drawings; and, in the second place, the accuracy of the drawings must be attested by mathematical or arithmetical calculations.

The word *series* is applied to that succession of related quantities each of which is formed in accordance with some fixed rule or principle. Each of the successive quantities is called a *term*. The fixed rule or principle is called the *law of the series.*

Theorem

The dimensions of all parts of the Parthenon may be mechanically determined by reference to three related series of squares.

Series I

By the law of the first series, each term or square is double the area of its predecessor. The

7. The Attic Foot and Attic Stadion—A Table of Base-Line Dimensions with Their Squares.

S	10,000.	×	10,000.	Sta. =	100,000,000.	Sq. Sta. =	31,250,000,000,000.	Sq. Ft.				
R					20,000,000.	Sq. Sta. =	6,250,000,000,000.	Sq. Ft. =	2,500,000.	×	2,500,000.	Ft.
Q	2,000.	×	2,000.	Sta. =	4,000,000.	Sq. Sta.	1,250,000,000,000.	Sq. Ft.				
P					800,000.	Sq. Sta. =	250,000,000,000.	Sq. Ft. =	500,000.	×	500,000.	Ft.
O	400.	×	400.	Sta. =	160,000.	Sq. Sta. =	50,000,000,000.	Sq. Ft.				
N					32,000.	Sq. Sta. =	10,000,000,000.	Sq. Ft. =	100,000.	×	100,000.	Ft.
M	80.	×	80.	Sta. =	6,400.	Sq. Sta. =	2,000,000,000.	Sq. Ft.				
L					1,280.	Sq. Sta. =	400,000,000.	Sq. Ft. =	20,000.	×	20,000.	Ft.
K	16.	×	16.	Sta. =	256.	Sq. Sta. =	80,000,000.	Sq. Ft.				
J					51.2	Sq. Sta. =	16,000,000.	Sq. Ft. =	4,000.	×	4,000.	Ft.
I	3.2	×	3.2	Sta. =	10.24	Sq. Sta. =	3,200,000.	Sq. Ft.				
H					2.048	Sq. Sta.	640,000.	Sq. Ft. =	800.	×	800.	Ft.
G	.64	×	.64	Sta. =	0.4096	Sq. Sta. =	128,000.	Sq. Ft.				
F					0 08192	Sq. Sta. =	25,600.	Sq. Ft. =	160.	×	160.	Ft.
E	.128	×	.128	Sta. =	0.016384	Sq. Sta. =	5,120.	Sq. Ft.				
D					0.0032768	Sq. Sta. =	1,024.	Sq. Ft. =	32.	×	32.	Ft.
C	.0256.	×	.0256	Sta. =	0.00065536	Sq. Sta. =	204.8	Sq. Ft.				
B					0.000131072	Sq. Sta. =	40.96	Sq. Ft. =	6.4	×	6.4	Ft.
A	.00512	×	.00512	Sta. =	0.0000262144	Sq. Sta. =	8.192	Sq. Ft.				

series is produced in geometry by using the diagonal of one square to form the side or root of the next term of the series. The series is expressed arithmetically as 1, 2, 4, 8, 16, 32, etc.

In the Parthenon, this series, in concentric placing, with unity having a fixed relation to the Attic foot, is used to determine the proportions of the sculpture as used in relation to architectural members, and in minor architectural features generally where complex specialization is not required.

Series II

By the law of the second series, the sides of the progressing squares or terms are in extreme and mean proportion. The sum of two successive terms produce the next term. Geometrically the series is produced as shown in Plate V. The method given in Euclid not being symmetrical, is not available in the arts. Arithmetically the series is expressed only approximately by the terms 1, 2, 3, 5, 8, 13, 21, 34, etc. As the series converges, each term is the product of its predecessor into the quantity or multiplier 1.618+. This series is commonly known as the "Golden Section" or Fibonacci series.

In the Parthenon, in the mouldings and other members of the cornice and entablature, in the stylobat, and in the capitals of the columns at the cincture and the annulets, in all of which members the lengths are indeterminate and only heights and depths of section are directly manifest, all quantities are in this series of the "Golden Section." See Plate V and Plate VI. In Plate II the northeast wall of the Acropolis is shown in its relation to this series. This same series is expanded into the wall of the city; but the scale of Plate III is too small to show the series without confusion.

This second series, the "Golden Section," is related to the first, or binary, series either by the method of Euclid as in the cella, Plate I, or by the method used in developing the triglyph from the metope, Plate V.

Series III

In all other parts of the Parthenon where mass (length, breadth, and thickness) is manifest, all parts are determined by a series of squares progressing in their areas in exact multiples of five. This series is developed geometrically by doubling, side by side, each term and using the diagonal of the combined two squares as the side or root of the next term of the series. As commonly stated, the diagonal of two squares is the square root of five, meaning the side of a square whose area is five times that of the area of one of the first two squares. Arithmetically the series, expressing the areas, would read: Unity, 5, 25, 125, 625, and so on. It is this series that extends to the Piræus.

The terms of the third series are related to the terms of the second series through the third term representing the axial distance of the peristyle, being derived from the doubling of the sum of the last two terms of the second series. See Plate VI. This series also has an oblique relation to the Attic foot, in that a rectangle whose end corresponds to one term and its side to the next term, commonly called "the root five rectangle," has for its diagonal a quantity that is commensurable with the Attic foot and with the stadion.

As a corollary of the above, inasmuch as these different progressions of squares are evolved one from another, geometrically, all parts of the structure are definitely and definably related to one another and to the whole, in the sense that, given one part, all other parts may be obtained from it by a simple, uniform geometrical or mechanical process, a process of filiation or descent. By *definable relationship* is meant that physical one in which a definite part of one compound or organism is taken from it, to form a definite part of another similar compound or organism. To apply this definition geometrically, one may consider that a compound shape, as a square, has, elementally, 4 sides and 2 diagonals. If, by means of a pair of dividers or compasses, or otherwise, we take the diagonal

of such a square and erect on it a square whose sides are all identical in length with that diagonal, we have then two squares related by a common element or quantity. We have obtained the relationship mechanically, without the use of scale, or of calculation, or reference to any other standard than the square first taken. We may continue the process indefinitely in outward expansion and inward contraction, if the squares are arranged concentrically, and thus construct an *area scale* of commensurable quantities. All parts are related to each other and to the parent square. We may refer to this last as the master square. This may be a square foot, or a square yard, or a square mile. In the metope of the Parthenon, the master square is related to the Attic foot. Its side is 1 and 1-64 Attic feet, or 332 millimeters.

It is this unit, expanding by means of successive diagonals and sides or roots of the squares, which is developed first in the metope, by Series I; next in the entablature, by Series II. Then, by doubling the terms of Series II, the progression is carried into Series III, and from there on to the finish.

If these drawings were started at the actual size of the master square and continued full size on a platform or base capable of receiving them, they could be used as working full-size details of a building that would correspond with the Parthenon in all its parts.

It should be distinctly understood that the writer is not attempting to formulate a method of design. What is presented is a scale of related quantities which were, or could have been, used by the designer of the Parthenon in the same manner that a composer of music uses a scale of mathematically related sounds: sounds that, because so related, are recognized as musical, but which, if they had been unrelated, would have been merely noise.[8]

To illustrate the freedom with which a designer can use this rigidly uncompromising scale, the writer can cite the Temple of Apollo at Bassæ. This was designed by Ictinus, the architect of the Parthenon. He used the same fundamental methods in each building, and, by a variation of the method of handling, obtained, especially in the interior, entirely unlike results. In the Erectheum, every detail of the plan, section and elevation and varying grades of the porches, has the same mechanical relationship between the parts and the whole, to the same extent as in the Parthenon. There is a unity in the plan of the Erectheum which negatives the contention of Dorpfelds that the original plan contemplated a symmetrical building. Rather it proves the contention of Wheeler that the building was built as originally planned.[9]

Another example of the application of the principle of the progressing squares is found in the Propylæa. The writer has not applied the mathematical, or arithmetical, tests to the dimensions of this building; but, if the drawings of Penrose are correct, the square and compasses show that, in principle, the method of design is the same as in the Parthenon. All of these buildings, so unlike in style and detail and plan, are alike in their fundamental principles. In each, every part is apparently related to every other part and to the whole, and the buildings themselves are related to one another, through having a common unit of measurement applied in the same or in a similar manner throughout.

It is this inflexibility of the progression, this mathematically precise formula, that makes the checking of the accuracy of the drawings an easy matter. It is not necessary to depend on super-accuracy of draughtsmanship to prove the results. All quantities may be verified mathematically, by modern universally used formulæ, or by the ordinary tables of square roots in the handbooks. These quantities are all, in the

8. Alfred Daniell, *Principles of Physics*, 1904, p. 417.

9. Charles Heald Wheeler, *Original Plan of the Erectheum*, A. J. of Archaeology, XXV, 2, p. 139. Dorpfeld, *Das Hekatompedon in Athen.* Jb. Arch., XXXIV, 1919, pp. 1–40.

Parthenon, properties of the multiples of two and five and their square roots.

It should be remembered that the term *square root* has contrary applications in arithmetic and geometry. In modern arithmetic, a *number* is first assumed and from this, by a devious arithmetical process, another number is extracted which multiplied by itself produces the first number. This is the arithmetical square root. In geometry, a lineal quantity is taken or assumed. This is the root of a square and the square is accordingly built up on, or it grows from, this root. Numerals do not appear in the process. It will be observed in the successive drawings that the root is first obtained by geometrical process and the square is then drawn from the root. The process is repeated five times in the Parthenon; and, during it, we are not concerned with numerals, except to compare quantities with the known measurements of the building for purposes of verification, or for descriptive purposes, or for checking accounts based on a standard of values.

Description of Plates

In presenting a series of drawings showing a progressing series of terms, the strictly logical method might be to show first the drawing illustrating the lowest term of the series. This method has, however, certain practical disadvantages. The fine proportions of the cincture and annulets on the capitals, or of the pearl and reed moulding over the triglyphs, while perfectly in proportion as members of the series, may well wait while larger matters are discussed. For this reason, there has been selected for the first plate, as being of major importance, the drawing showing the entire third series of terms of the Parthenon, from the capital of the columns to the length of the cella, with the appurtenances of peristyle, posticum, pronaos, and stylobat. The series is continued in Plate II along its well marked course through the Acropolis, and in Plate III through the city of Athens to its wall and along the walled and fortified corridor to the Piræus. Plate IV, showing the posticum with the elevation of the frieze and the plan of the coffered ceiling, is shown next for the reason that the proportions of the posticum are the proportions of the port as laid out in its city planning.

Plate I

The series starts with the square marked "*A*" representing the height of the capital of the peristyle columns. The reader will refer to Plate VII for details of this member. The square "*A*" is doubled in area by putting a half square at either side to maintain symmetry of its axes. The diagonal of the two squares is then rotated, or pivoted, on the centre of the original square, still for the purpose of maintaining the symmetry and keeping the sides of the succeeding squares parallel. This rotated diagonal is the side, or root, of a square five times the area of the first square, as proved by the familiar Pythagorean proposition. The square is then completed outwardly from the axes and concentric with square "*A*," and becomes the second term of the series, or square "*B*." Square "*B*" exactly encloses the twenty-sided polygon of the northwest angle column. See Plate VII and Plate XI.

Proceed with square "*B*" as with square "*A*" to produce the square, or term, "*C*." The side of square "*C*" is the axial distance of the columns of the peristyle. Its area is five times the area of "*B*" and twenty-five times the area of "*A*." Its side, therefore, is commensurate with, and five times the length of, the side of "*A*." Its detail is shown in Plate VIII. These detail plates will be later described.

Proceed with square "*C*" as before to obtain square "*D*." This square is the column height when augmented by the rise of the steps. Reference to Plate IX will show the precise relation of this square to the cinctures of the columns of the east front.

Proceed with square "*D*" as before to obtain square "*E*," the fifth term in the series. This

square exactly determines the width of the cella, over all. It is commensurate with square "*C*," being twenty-five times its area and having sides five times the length of the side of "*C*," thus furnishing the destinations of the six columns on either end of the cella having five equal axial spacings. The other two columns, the angle columns, instead of having their central axes placed on the uniform spacing of the other columns, have *the outer faces of the neck of the columns placed on the interval*, as shown in Plates X and XI. This square "*C*" placed along the sides of square "*E*" gives the longitudinal destination of the necks of the columns along the flanks of the peristyle.

In the same manner, the square "*C*" is repeated along the tops of the columns, and limits the height of the entablature. In this connection, the capitals are considered as parts not of the columns but of the entablature, the bottom of the architrave being thus relatively unimportant. See Plate VI and Plate X.

Proceed with square "*E*" as before to produce square "*F*." This square, while apparently the length of the cella, is actually over three-tenths of a foot shorter in a length of approximately 159 feet. There is an apparent explanation of this discrepancy, the only one in the series that is not negligible, in the fact that this is the only way in which the axial column spacings of the ends of the building could be repeated on the flanks. As it is, in spite of the lengthening of the building, the columns are slightly closer on the flanks than on the ends, although only by about two hundredths of a foot.

The half of square "*E*" placed on the end of the cella and the half of square "*D*" placed on the flanks furnish the projection of the mean line of the cornice, entablature, and profile of the inclined columns. This projection is shown, in the plan, in a dotted line just outside the line of the top step. This is shown in detail in Plate XI.

Having found the projection of the necks of the columns on the flanks by means of square "*C*" applied to square "*E*" and having thus established the relation of the necks to the mean line of the cornice, we find the location of the line of necks of the end columns by means of the square mitre on the angles.

The location of the cross partition is determined by laying off the inside width of the cella along the inside wall of the flank of the naos; in other words, by "squaring it." The half distance is at point "*P*." With radius "*P*"–"*W*," draw the arc as shown whose termination marks the location of the foundation block of the cross wall.

The wall itself seems to align on its east face with the necks of the peristyle columns as indicated. This method of alignment is shown also on the columns of the pronaos where at the necks the alignment is perfect with the opposite necks of the columns of the peristyle.

The whole geometric series seems to attain its perfect harmonies only at the level of the capitals. The apparent confusion occurs only at the pavement level where the bases of the columns are spread, and the walls inclined from the vertical.

Another feature of the planning seems to be the use in the foundation blocks, or orthostate blocks, which form a projecting base for the walls, of an even multiple of the Attic foot, whereas the squares in the geometric series can be expressed on the diagonal of the cella very nearly in even multiples of the Attic foot. The discrepancies are taken up or rendered unnoticeable by the projections of the foundation or base courses from the faces of the walls. The nave in the naos measures on the floor exactly thirty Attic feet, divided across into six pavement blocks each five feet long. Between base blocks across the entire cella, the distance is exactly fifty-eight Attic feet. The extreme over-all length of the bottom step is two hundred and twenty Attic feet. It is therefore a fair generalization to say that, in their designs, the architects

achieved their harmonies of proportion by means of the square and compasses, entirely inexpressible in terms of any standard of measurement when taken on the square, but alternately commensurable with Attic foot and Attic stadion when taken on the diagonal of a rectangle which may be designated as a "root five" rectangle. These incommensurable dimensions were built on such foundations, approximating them, as could be laid off and specified in terms of the Attic foot or fractions of it. The divergencies were reconciled in a practical manner by taking advantage of the offsets of these base blocks and by inclining the walls and columns from the vertical, where such inclinations served an æsthetic or other purpose.

Plate II

The drawing is unavoidably made complex by the inclusion in it of the apparently older system of measurement carried out on the base line of an older Erectheum. This seemed to have been terminated at the altar place or Pnyx. The position of the marble slabs shows how the new base line and axis of the Parthenon were related to the other line. It is impossible to put further lines of relationship in the drawing without making it unnecessarily confusing, but it may be truthfully said that, should the Parthenon be entirely razed for the third time, but these markers left, it would be possible to reconstruct the building practically as it was before, and upon the same spot. It seems hardly possible, or in keeping with the methods of surveyors handed down to this day, that these slabs and monuments or markers were left uncovered, or at least undisguised, in ancient times.

In the walls of the Acropolis appears the extreme and mean ratio series, which is followed down the northeast wall and out into the wall of the city itself. The explanatory diagram in the corner gives the Euclidean method of drawing this series. This method was used in the naos to determine the cross wall of the cella. It differs from the method shown in the metope and triglyph only in being drawn without the axis of symmetry.

In the map of the Acropolis, the stadion appears as a whole unit. If Penrose is correct in his survey, it is marked by a stone wall, distant one stadion from the centre of the statue of Athena. Again the same distance can be "stepped off" with the beam compasses from points along the wall marked by buttresses or angles to other points and buttresses. This standardization of the stadion unit appears again strikingly in the map of the city wall where seven stadia is the standard unit, and appears again from point to point around the wall, in the distance from the centre of the Parthenon to the point of the east salient, and from that point right and left to points on the wall. These can be easily picked up by a pair of dividers and are not shown in the drawing because of their complexities, if all were shown.

Plate III

In the maps of the city and walls of Athens and of the Port, the terms of the progressing series are plainly marked, not alone by the stone platform but by the location of the temples, stoas, cross roads, gates, and angles of the wall. The horologium is placed on square "*I*" on the diagonal that passes through the well marked on the drawing of Penrose. The altar place on the Pnyx and the Temple of Jupiter are on square "*J*." The square "*K*" is marked on the east by the Stadium and on the west by the angles of the vestibule to the long passage to the port. This passage itself in its width between the walls is half the diagonal of square "*H*."

A careful search of the city and its environs to-day might, if guided by the squares, disclose still other markers. Particular search should be made along the prolongations of the sides of squares "*L*" and "*K*," one for the marker in the vicinity of the eight stadion standard gauge and further out the ten thousand Attic foot standard. On square "*L*," it should be possible to check

up on the line prolonged from the stone platform beyond the Piræus, to and through the Island of Stalida. Again it may be said that, should every mark be effaced from the Acropolis, all could be restored by means of the distant markers and base lines on the sea and on the land, and with the key furnished by the law and the terms of the series.

Drawn, with dotted lines, in Plate III is an outline plan and elevation of the Parthenon. It is enlarged from term "*F*" to term "*M*," or to the seventh power of the square root of five. What was four Attic feet in the lower power is now two stadia—this owing to the fact that the foot and the stadion are also terms in the identical series of geometrical progression. The drawing shows that the entrance port of the city is in plan the same as the entrance to the Parthenon, raised to the seventh power of the square root of five. Here again the diagonals, the steps, and the columns, with walls and pilasters, are marked by the locations of havens, city streets, water gates, gates in the walls, propylæa and temples, and stone platforms to mark the prolongations of the axes.

Whether there remain, to-day, markers on the Island of Stalida or on the Mountain of Pausanius or on Mount Hymettus, the writer cannot say. But the position of the water gate to the larger haven, with the axis of its propylæa and the corner of the temple beyond, suggests a marker on the Mountain of Pausanius to the north and another on Mount Hymettus to the east.

Plate IV

Of the columns of the posticum, with the sculptured frieze and the coffered ceiling, what may be said? They in their combination, together with the maps of Curtius and Milchöffer, bring home the poignant truth that the study of the Parthenon and of Hellenic culture is only in its beginning, and the writer submits this drawing without comment to his fellow architects and to those sculptors who seek a method of relating their plastic art to the architectural composition it is to adorn.

Technical Description of Detail Plates

Plate V

The exact member in the Parthenon commensurable with the unit of measurement, is found in the one metope to the left of the central axis of the east front. See Plate V and Plate VI. The width of this metope divided by four, arithmetically, is the side of the unit square, or master square. This is equivalent to 332 millimeters. In geometrical development, this process is reversed. The unit square is assumed; and, by means of its diagonal, the progression is started which consummates in the square of the metope, whose area is sixteen times the area of the unit and whose side or root is four. See Plate V. Follow the line of arc "*T*" to "*U*" and of arc "*V*" to "*W*," etc., until the square b–b' or the metope is produced.

The triglyph is developed from the metope by drawing arcs a–b and a'–b' from centres w''' and w' with radii $w'''-b$ and $w'-b'$. Note that the radius is the diagonal of two unit squares or the root of five unit squares (square root of five). The side of the metope square is the height of the architrave. The heights of both metope and architrave are augmented as shown at "*x*"–"*y*." The inclination of the face of the entablature tends to diminish the width as seen from below. The expedient shown in "*x*"–"*y*" compensates for this diminution. The inclination of the face of the entablature is not perceptible to the eye looking in direct elevation; but it is strikingly apparent at the angles when seen in profile. The drawing of the square and the two half triglyphs is the same geometrically as the method of Euclid in establishing the extreme and mean ratio, with the important exception that the increase or decrease is arranged symmetrically on two axes. Using the metope width as the

maximum of a series and subtracting either with the dividers or arithmetically the width of the triglyph from it, a third member of the series is found, and so on to the fourth and continuing until the compass becomes too blunt an instrument to record the minute differences. It will be found by reference to the drawings, Plate V, Plate VI, and particularly to the detail angle column, Plate XI, that there have been laid down in this progression all of the dimensions of the cornice and entablature and the other members in Series II of the theorem.

Plate VI

In Plate VI, the side of the rectangle *a–a′* obtained from Plate V, is shown doubled. To review the process, the binary ratio Series I reached its consummation in the metope square. The sum of the metope and the triglyph marked the consummation of the extreme and mean ratio, which was itself evolved from the binary ratio by the doubling of two of its units and adding the diagonal of these units as shown in Plate V. Now in turn the consummation of the extreme and mean ratio of Series II is doubled to form the side of a new square which is a member of a new series in this promorphology, a series of squares increasing in multiples of five, or decreasing by segmentation into units of five. We have here an evolution of mechanisms, and a geometrical heredity which suggests a foundation for some of the vague traditions of Vitruvius in regard to the design of the Parthenon: traditions which it will be unnecessary to discuss here, dealing as they do with questions of biology and the derivation of the ratios.[10]

The root or side of square *C–C′*, Plate VI, is the doubled quantity *a–a′*. One side of this square is segmented into five equal portions, and one of these segments is taken as a new unit for the progression in series of fives. Note, at the bottom of Plate VI, the graphic or geometrical method of increasing the unit square in multiples of five. This method in its successive steps furnishes the key to the entire development of quantities as laid down in Series III of the theorem. While it is not essentially different from the method of Euclid, it does differ in this detail: it is concentric and on axes of symmetry. It is thus directly, not indirectly, applicable to the uses of symmetrical design.

Plate VII

In Plate VII, the unit square of Plate VI is increased to form the square *B–B″* of five units. The side of the unit square (No. 1) is the height of the capital. The plan of the column, at the cincture height, is drawn with its axis coincident with the top of the abacus. The radius of the column is derived from the diagonal of the square *f–f′*, Plate V. Note that the facets of the polygon in the plan determine the location of the heights of the echinus and annulets in the elevation. The squares on the lines *PO* and *RQ* determine the width of the abacus. The origin and destination of the tangent lines of the echinus and annulets are determined by the facets in the plan of the base of the column, which base was determined by the square *B–B″*. Note that the square No. 1 is placed in its entirety on the end of the rectangle in Plate VII, while, in the following plate, it is divided half and half; the first being in elevation and on one axis of symmetry and the second being in plan and on two axes.

Plate VIII

In Plate VIII, the square *B–B″* is increased to form the side of square *C–C″*. This is the square before derived from the doubled metope and triglyph shown in Plate VI. This square determines the axial distance of the columns. On the lower margin, the diagrams show how the reciprocals are applied in the ground plans on two axes of symmetry.

10. See H. W. Conn, *Method of Evolution*, 1903, pp. 163–169. Ernst Haeckel, *History of Creation*, I, p. 348. D'Arcy W. Thompson, *Growth and Form*, Cambridge, 1917.

Plate IX

The square C–C'' of Plate VIII is carried forward to form the unit square of Plate IX. The side of the square D–D'' is the column height when the height of the individual columns is augmented by the rise of the step from the datum line. This height, so increased, is recognized by Penrose as the real height, in reason, when proportion is being established with the building as a whole. It suggests a reason for the augmented master unit, 1-64 of an Attic foot above the normal foot. If the normal foot were increased geometrically as the master square has been increased, it would determine the individual heights of the column.

Plate X

The process of increase is repeated in Plate X to form the side of square E–E'' or Square No. 5, which contains 625 units. This square determines the width of the cella. The sum of the two squares D–D'' determines the inside dimension of the cella. The difference between the sum of the D–D'' squares and the side of E–E'' determines the thickness of the flank walls, NS and RM. Square E–E'' is carried forward to form the square of the cella length shown in the lower half of this plate.

In the lower half of Plate X the plan of the Parthenon is developed precisely as in the preceding plates. The square F–F'' is the determining factor in the length of the cella. The square on the end is divided into the pronaos and posticum. The square on the reciprocals D–D'' is divided to form the depth of the peristyle along the flanks. This square also determines the position of the row of columns inside the cella, in the naos. See "X" on Plate X. The depths of the porches and the over-all dimensions are all referred to the line "W–Y" shown on Plates VI, X, and XI, which indicates the mean line of the profile of the column and entablature, or one-half the total distance from the utmost recession of the neck of the columns at the angles (at the cinctures) to the projection by plumb line of the cymatium, Plate XI. That this was done consistently and logically is shown by the fact that the step at the foot of the inner row of six columns in the posticum is exactly centered between this imaginary or axial line and the face of the cella wall. See plan of posticum, Plate IV. It will be seen, by referring back to Plate VII, how little the location of the steps influences the design, and how unimportant they are in the general scheme.

The net length of the two chambers of the cella, the naos and opisthodomos, is determined by the sum of the two squares, E–E'', exactly as the width was determined by the squares D–D'' in Plate X. The difference between the sum of the two squares, (E–E' plus E–E') and the side of F–F'' is allotted to the thickness of the transverse walls. In the cross section of the cella, there are only the two flank walls to be considered. In the longitudinal section, there are the two end walls and a cross partition to take up the allotment. The method of making the allotment is shown on the plan at N and M. The location of the partition wall is obtained by squaring the inside width of the naos ($MNOK$). Bisect the side MK at L. With the line LN as radius, describe the arc NOH. H is the point of origin of the cross partition. MN and MH are in extreme and mean ratio. This ratio is disguised by the thickness of the cross wall, which is subtracted from the rectangle, instead of added to it. The intersection of the main axis of the building with the side of the square KO locates the statue of Athena.

The drawings do not show the roof plan, but each slab is composed of a "root five rectangle" with a square at either end. The elevation of the flanks would show five squares, the side of each square being the sum of the sides of D–D'' and C–C''. C–C' is the axial distance of the columns. Let C–C' equal 1. Then C–C' plus D–D' equals 3.236. The distance across the front would be 7.236, or seven axial distances plus .236 of this

distance. On the flanks, 5 times 3.236 equals 16.180, or sixteen axial distances plus .180 of this distance. Note the dilemma. If the axial distance is to be the same on fronts and flanks then the fractional remainders allotted to the projections of cornice would be unequal in the proportion of .180 to .236. The builders made use of a compromise. The building was lengthened to what amounts to 0.4 English foot. This left the cornice allotment, of course, equal on front and flanks and the column spacings practically identical on all four façades. In the development of the flanks, the familiar extreme and mean ratio (1.618) is exhibited multiplied by 10.

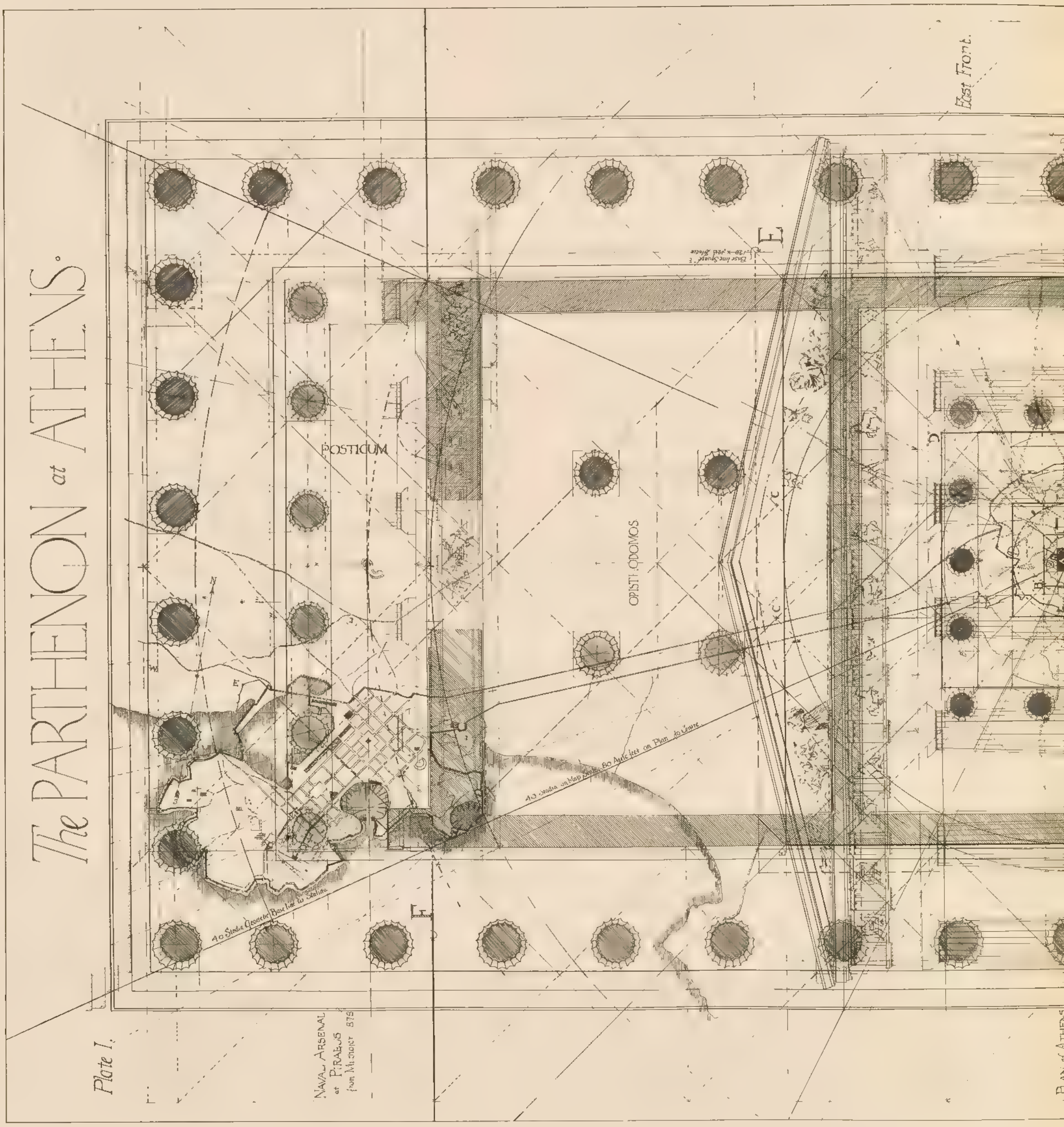
Plate XI

Plate XI is a detail sheet and not a part of the progressing series. It is a preliminary study showing the step by which the method of expression by the unit system was finally developed. It is arithmetical as well as geometrical in its method of presentation. The original drawing is made one-third full size and by the accuracy thus obtainable many obscure relations were brought to light. It is the last in point of position but was the initial drawing in the series of studies.

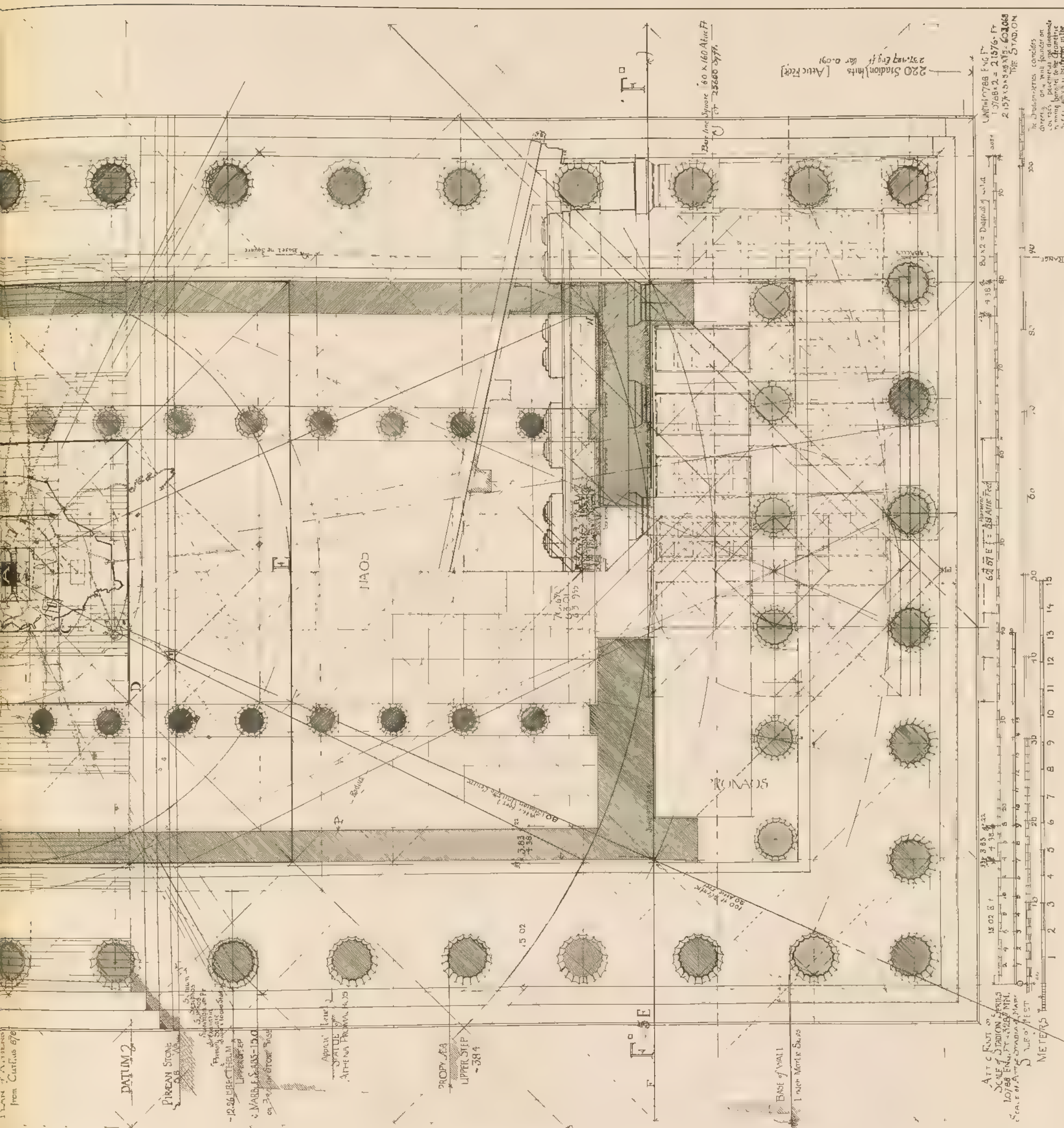
Conclusion

After following the development of the geometrical progressions step by step and comparing this development with the measurements of the Parthenon as given by Penrose, the reader will form his own conclusions as to whether art or science predominates in the designing of the Parthenon, or whether, as the present writer believes, they go hand in hand. Whatever the conclusions are as they may affect the realm of art, in the realm of science it will be seen that the theorem stated at the opening of the constructive portion of this paper has been demonstrated.

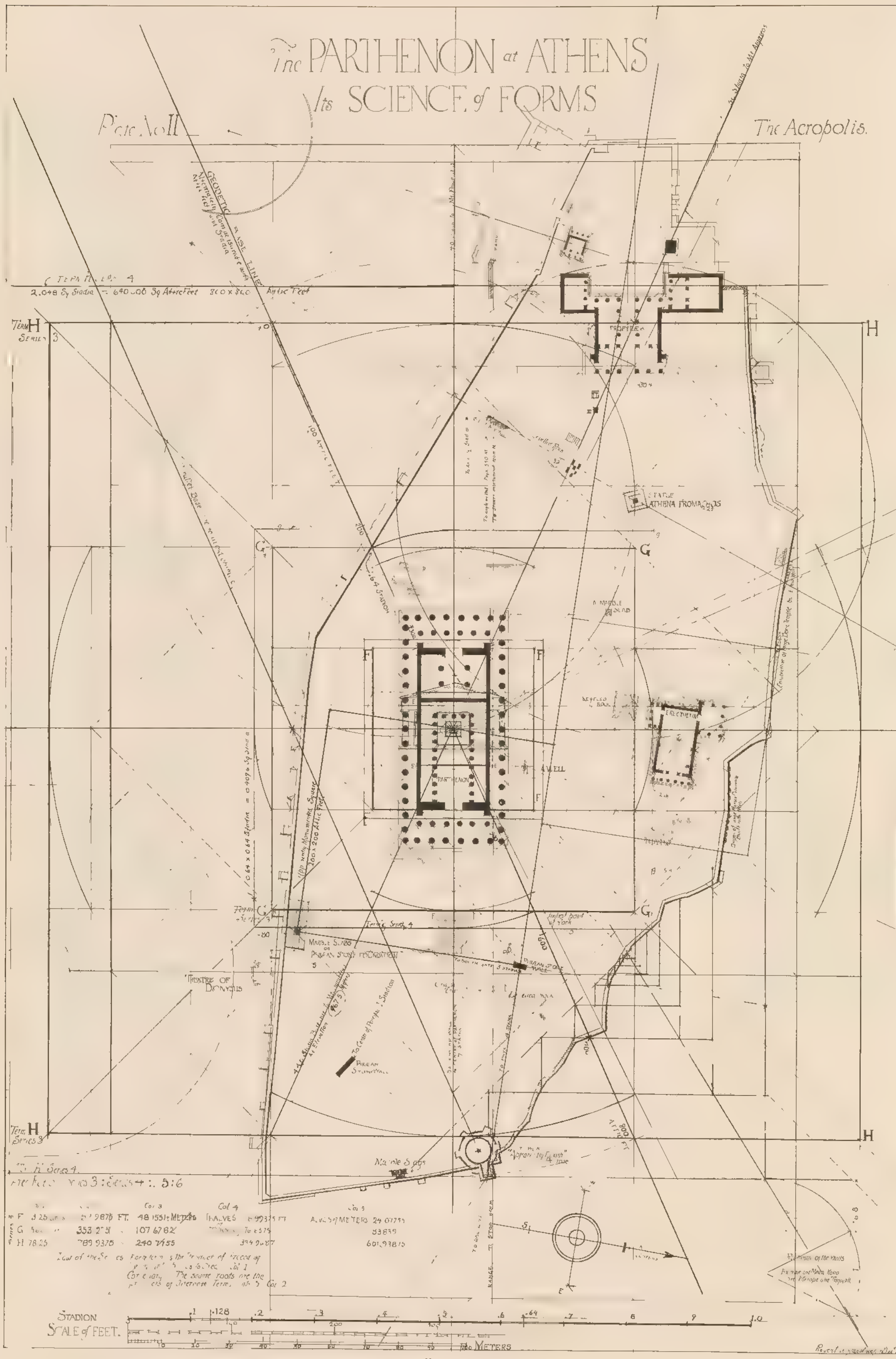
PLATES



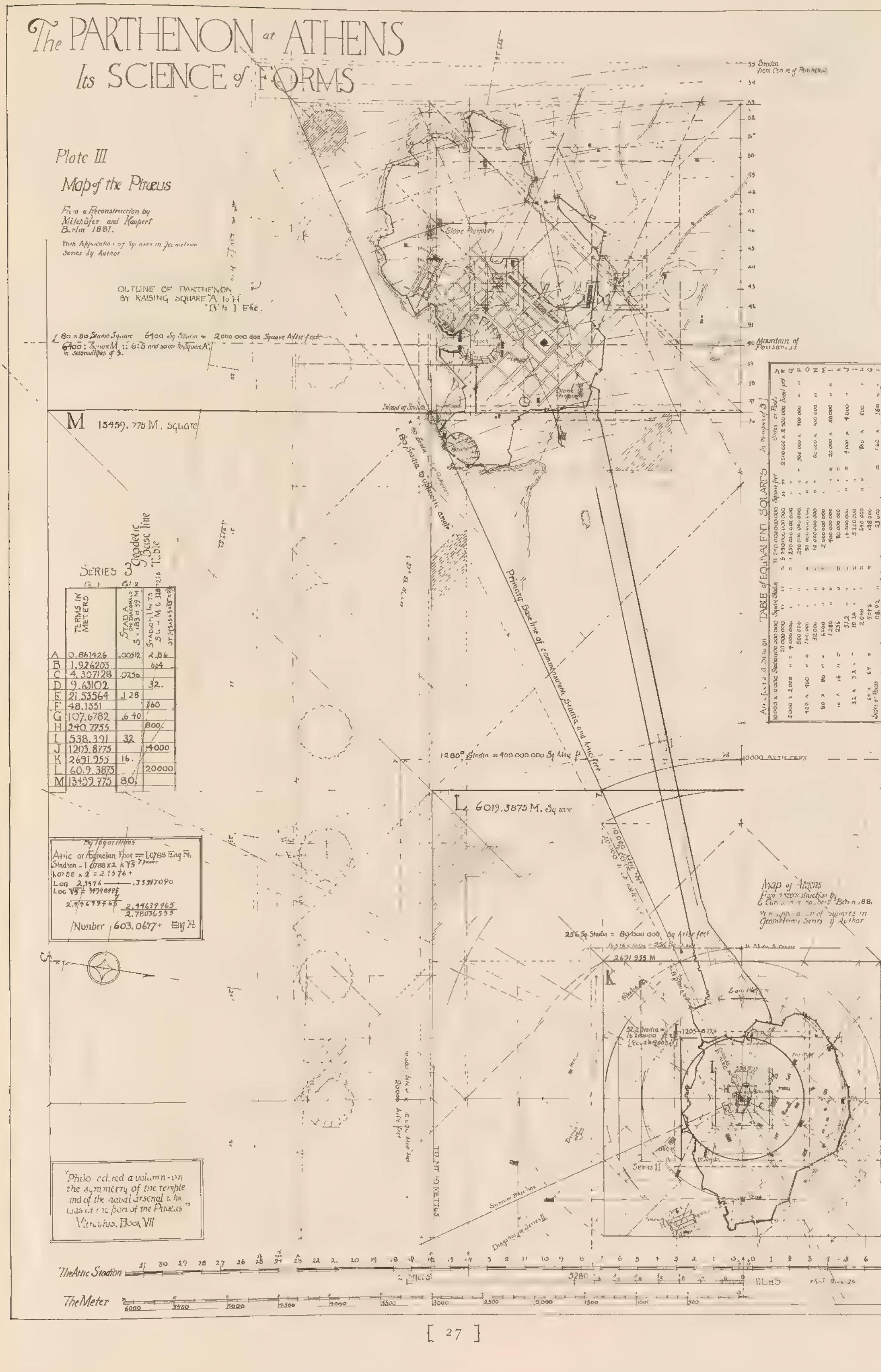
The PARTHENON at ATHENS.
Plate I.
East Front.
POSTICUM
OPISTHODOMOS
NAVAL ARSENAL at PIRAEUS



NAOS
PRONAOS
DATUM 2
BASE of WALL
METERS



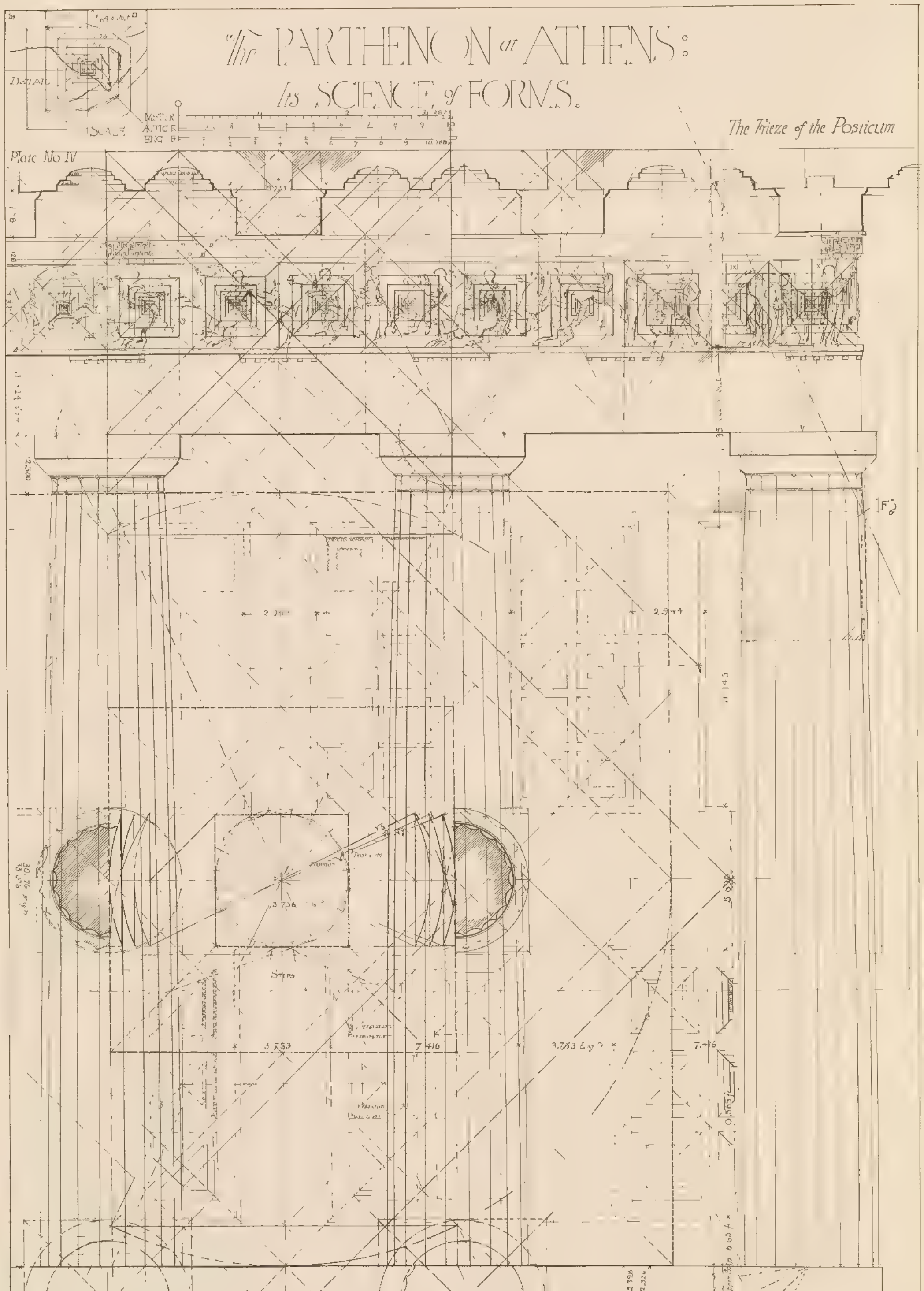
The PARTHENON at ATHENS
Its SCIENCE of FORMS
The Acropolis.
H
G
F
PROPYLÆA
STATUE
ATHENA PROMACHOS
PARTHENON
ERECHTHEUM
THEATRE OF
DIONYSUS
STADION
SCALE of FEET.
METERS



The PARTHENON at ATHENS
Its SCIENCE of FORMS
Plate III
Map of the Piræus
From a Reconstruction by Milchöfer and Kaupert Berlin 1881.
With Application of Squares in Geometrical Series by Author
OUTLINE OF PARTHENON BY RAISING SQUARE "A" to "H" "B" to "I" Etc.
80 x 80 Stadia Square 6400 Sq Stadia = 2000 000 000 Square Attic feet
6400 : Square M :: 6:5 and so on to Square A, in submultiples of 5.
M 13459.775 M. Square
Island of Stalida
Stone Platform
Mountain of Pausanias
55 Stadia from Centre of Parthenon
Primary Base line of commensurate Stadia and Attic feet
1280 Stadia = 400 000 000 Sq Attic feet
10000 Attic feet
L 6019.3875 M. Square
SERIES 3 Geodetic Base line Table

	Col 1 TERMS IN METERS	Col 2 STADIA ON DIAGONALS S. 183 d 39 M	STADIA (IN TS)
A	0.861426	.00512	2.56
B	1.926203		6.4
C	4.307128	.0256	
D	9.63102		32.
E	21.53564	.128	
F	48.1551		160
G	107.6782	.640	
H	240.7755		800
I	538.391	32	
J	1203.8775		4000
K	2691.955	16.	
L	60.9.3875		20000
M	13459.775	80.	

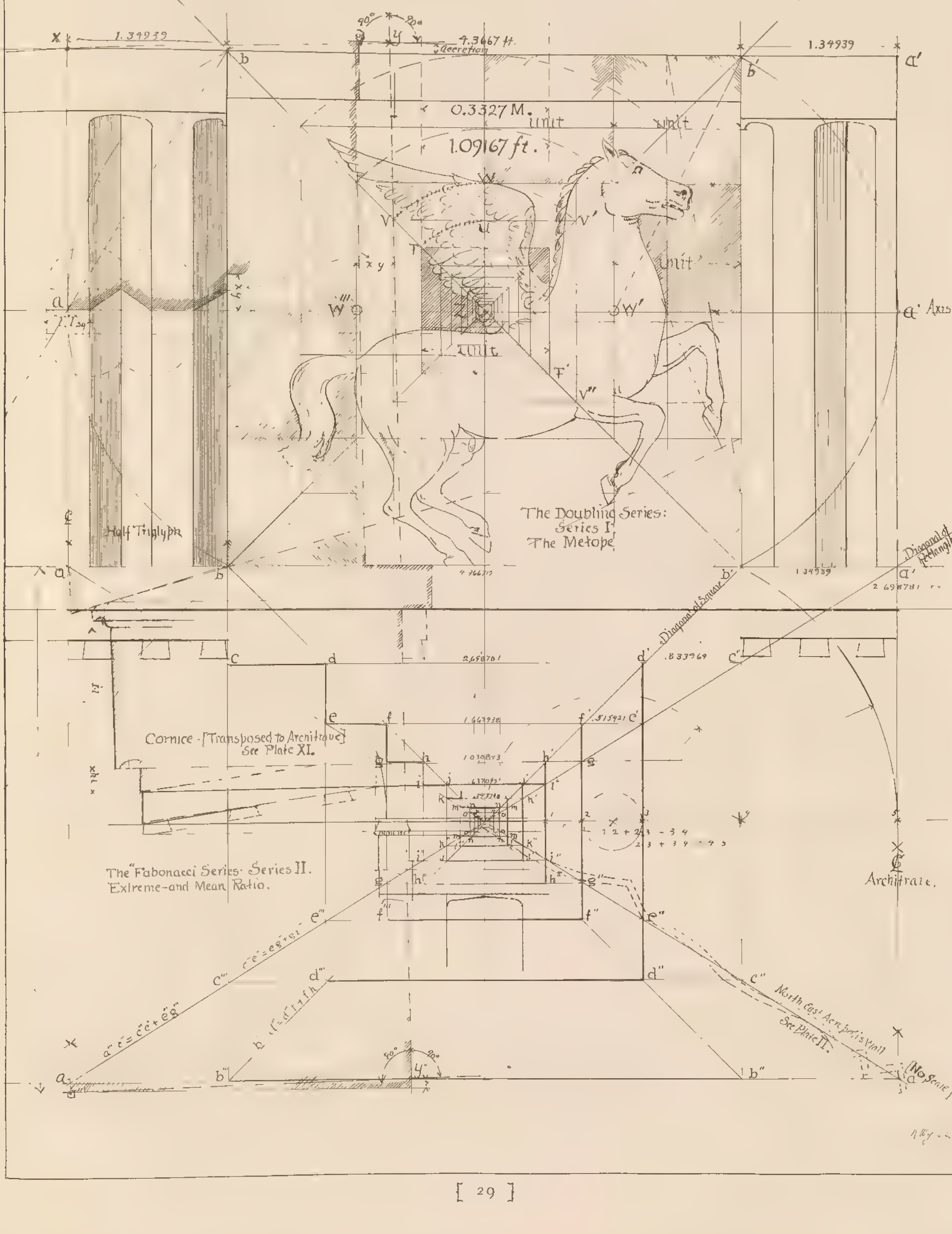
By logarithms
Attic or Æginetan Foot = 1.0788 Eng Ft.
Stadion = 1.0788 x 2 x √5 7 power
1.0788 x 2 = 2.1576 +
Log 2.1576 .33397090
Log √5 = .34948995
2.44639465
2.44639465
2.78036555
Number 603.0677+ Eng Ft.
TABLE of EQUIVALENT SQUARES
Map of Athens
From a reconstruction by Curtius and Kaupert Berlin 1881.
With Application of Squares in Geometrical Series by Author
256 Sq Stadia = 80 000 000 Sq Attic feet
2691.955 M
K
1203.8775
Series II
"Philo edited a volume on the symmetry of the temple and of the naval arsenal which was at the port of the Piræus"
Vitruvius. Book VII
The Attic Stadion
31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 3 2 1 10 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6
MILES
5280
The Meter
6000 5500 5000 4500 4000 3500 3000 2500 2000 1500 1000 500


The PARTHENON at ATHENS:
Its SCIENCE of FORMS.
The Frieze of the Posticum
Plate No IV
Steps
C. C. Cd 14.086

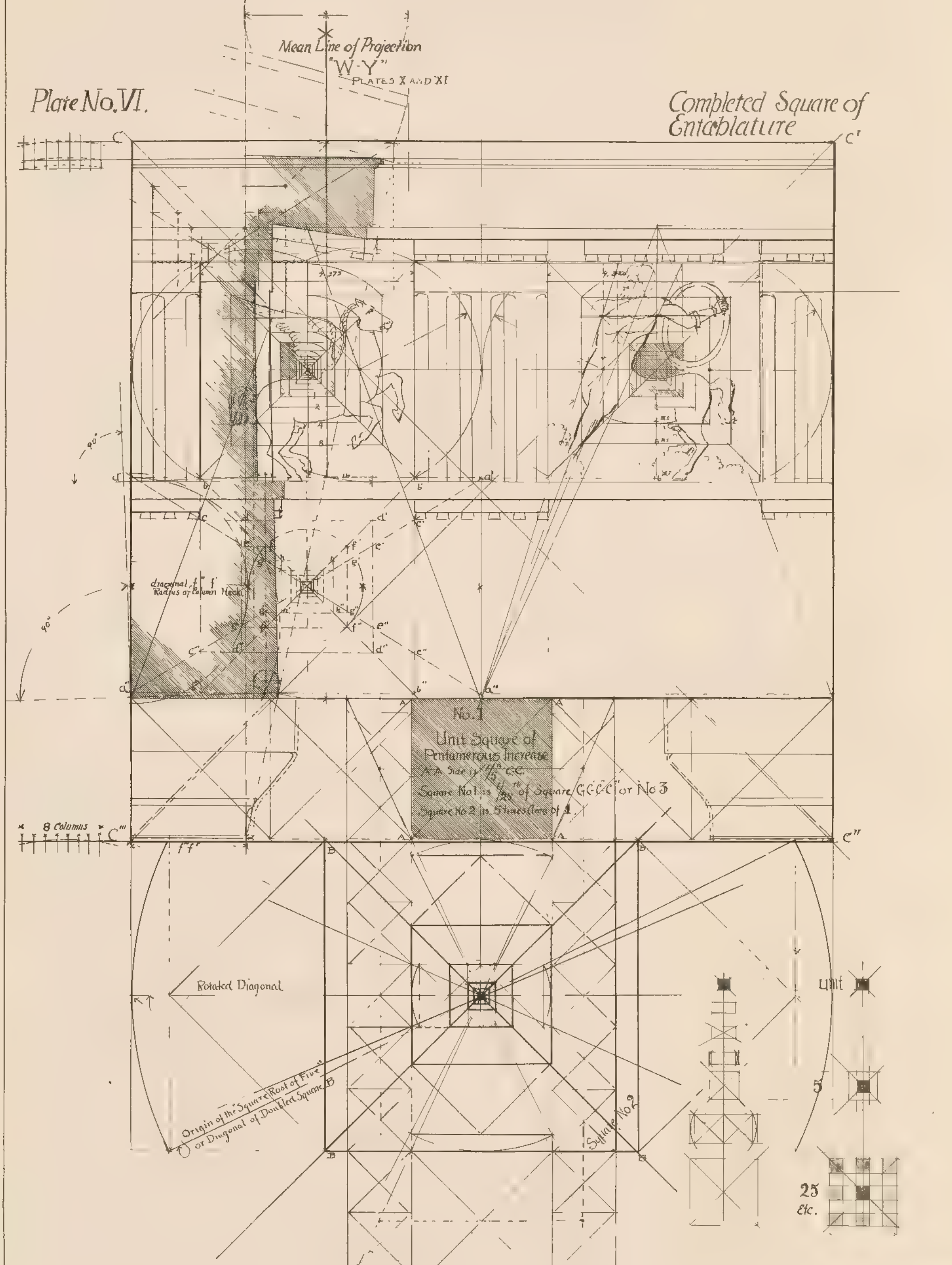
The PARTHENON at ATHENS:
Its SCIENCE of FORMS.

Plate No. V.

The Entablature.

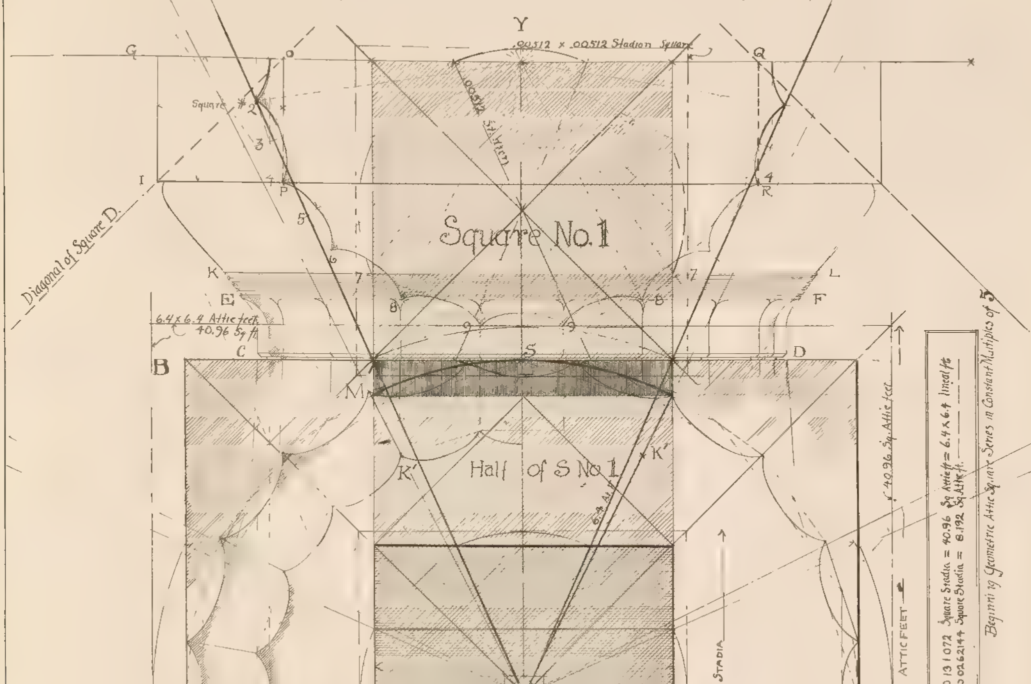


The PARTHENON *at* ATHENS:
Its SCIENCE *of* FORMS.



The PARTHENON *at* ATHENS:
Its SCIENCE *of* FORMS.

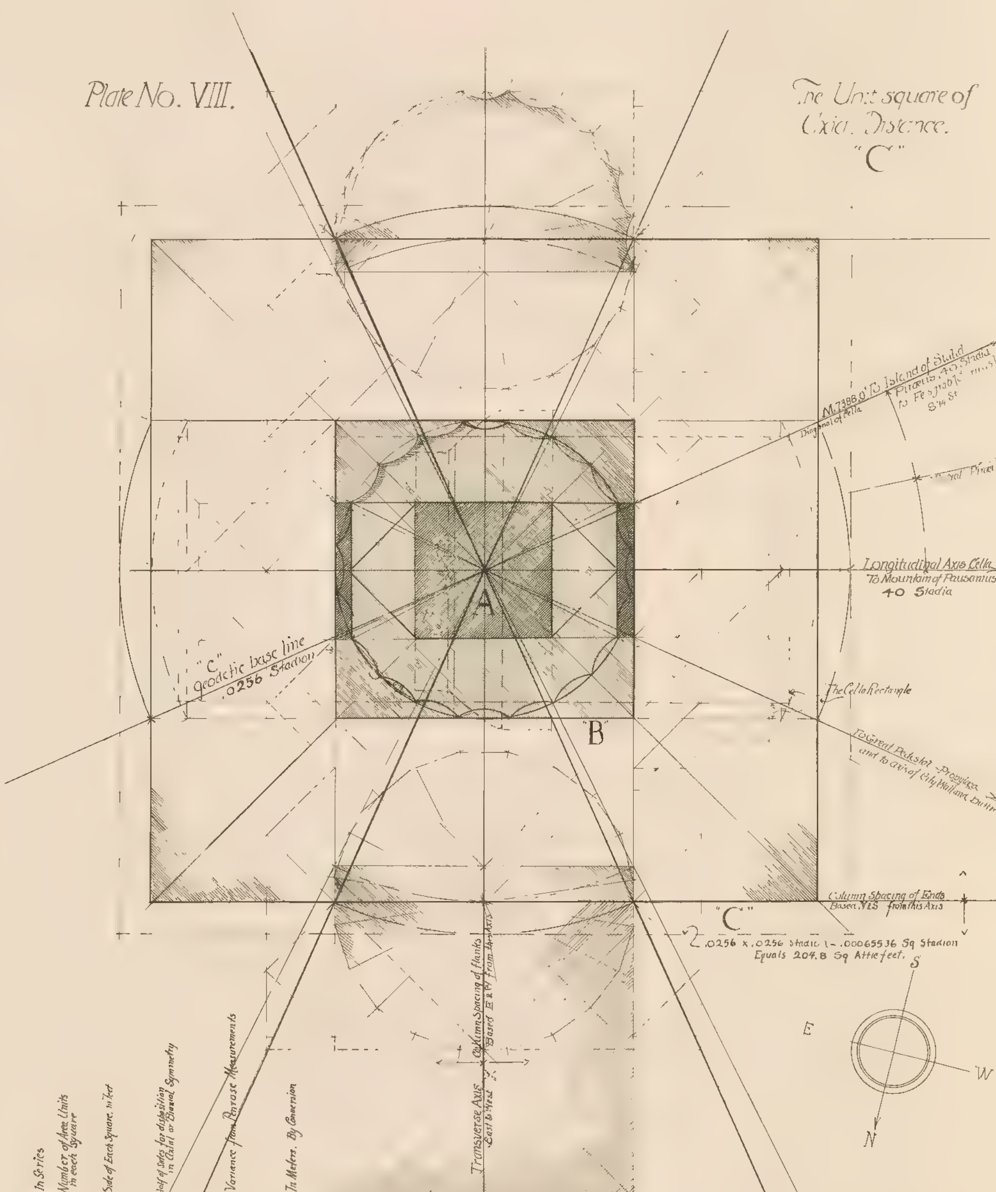
Plate No. VII. *The Unit of the Capital.*



Geom. Progression of Squares in Multiples of 5, in Area, in Concentric Destinations.

The PARTHENON *at* ATHENS:
Its SCIENCE *of* FORMS.

Plate No. VIII.



In Series	Number of Area Units in each Square	Side of Each Square, in Feet	Half of Sides for disposition in Axial or Biaxial Symmetry	Variance from Penrose Measurements	In Meters. By Conversion	
A	1.	2.8262+	1.4131	0.007	M. .86142	M. .86 [Collignon]
B	5.	6.3195+	3.15975	0.0095	1.926203	" 1.93 N.W. Col.
C	25	14.131	7.0655	0.00	4.307128	" 4.29 [Collignon]

"A" .00512 x .00512 Stadia	= .0000262144 Sq. St.	= 8.192 Sq. ft.		
"B"	.000131072	" " = 40.96	" " =	6.4 x 6.4 lin. ft.
"C" .0256 x .0256 "	= .00065536	" " = 204.8	" "	

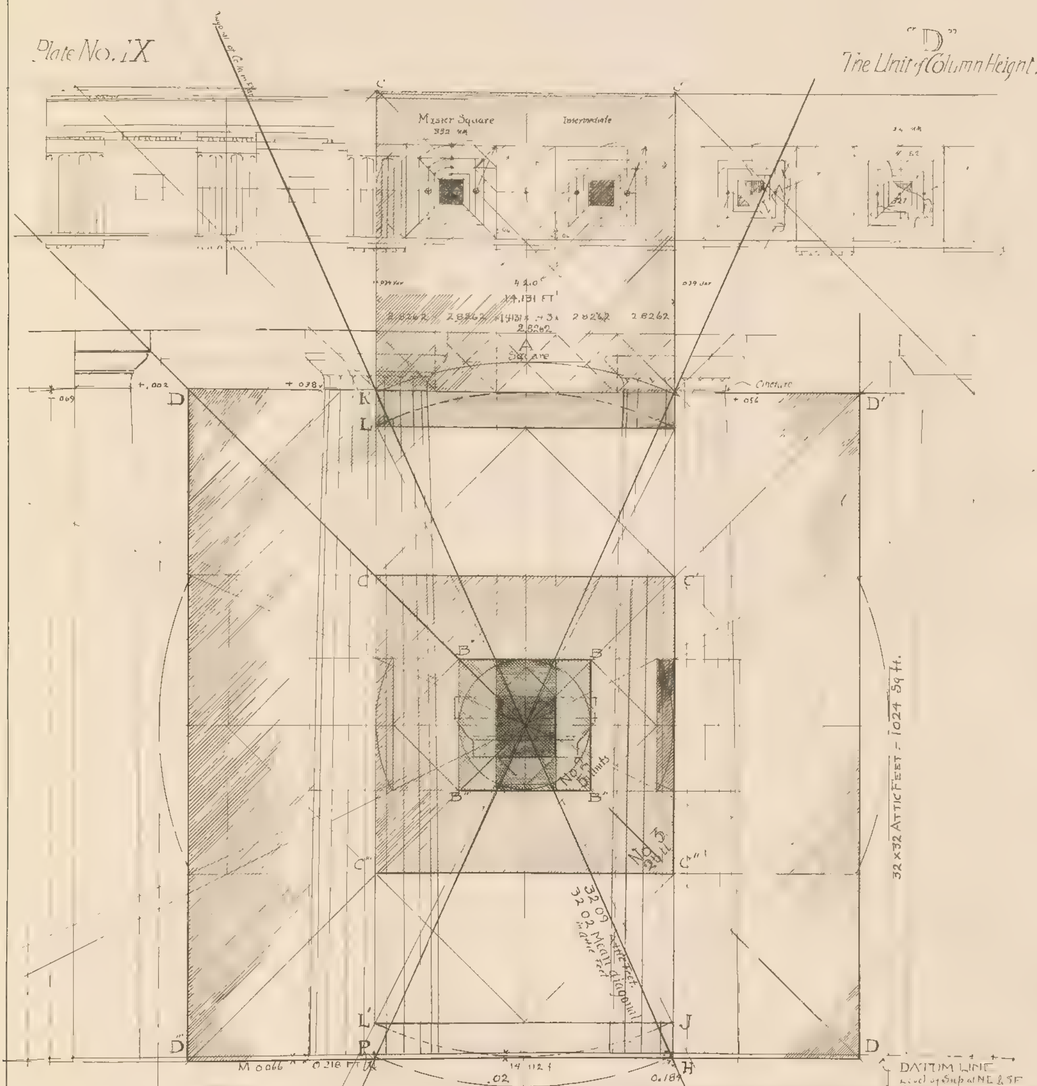
Area of Squares on the Axes is to A. of Squares on Diagonals as 5 is to 6.

R.W.G. Del.

The PARTHENON *at* ATHENS:
Its SCIENCE *of* FORMS.

Plate No. IX

"D"
The Unit of Column Height.



"A"	1	2.8262	1.4131	.007 ft	861426	.0
"B"	5	6.3195	3.15975	.0095	1.926203	.006
"C"	25	14.131	7.0655	.00	4.307128	.014
"D"	125	31.5975	15.79875	.014	9.63102	.0

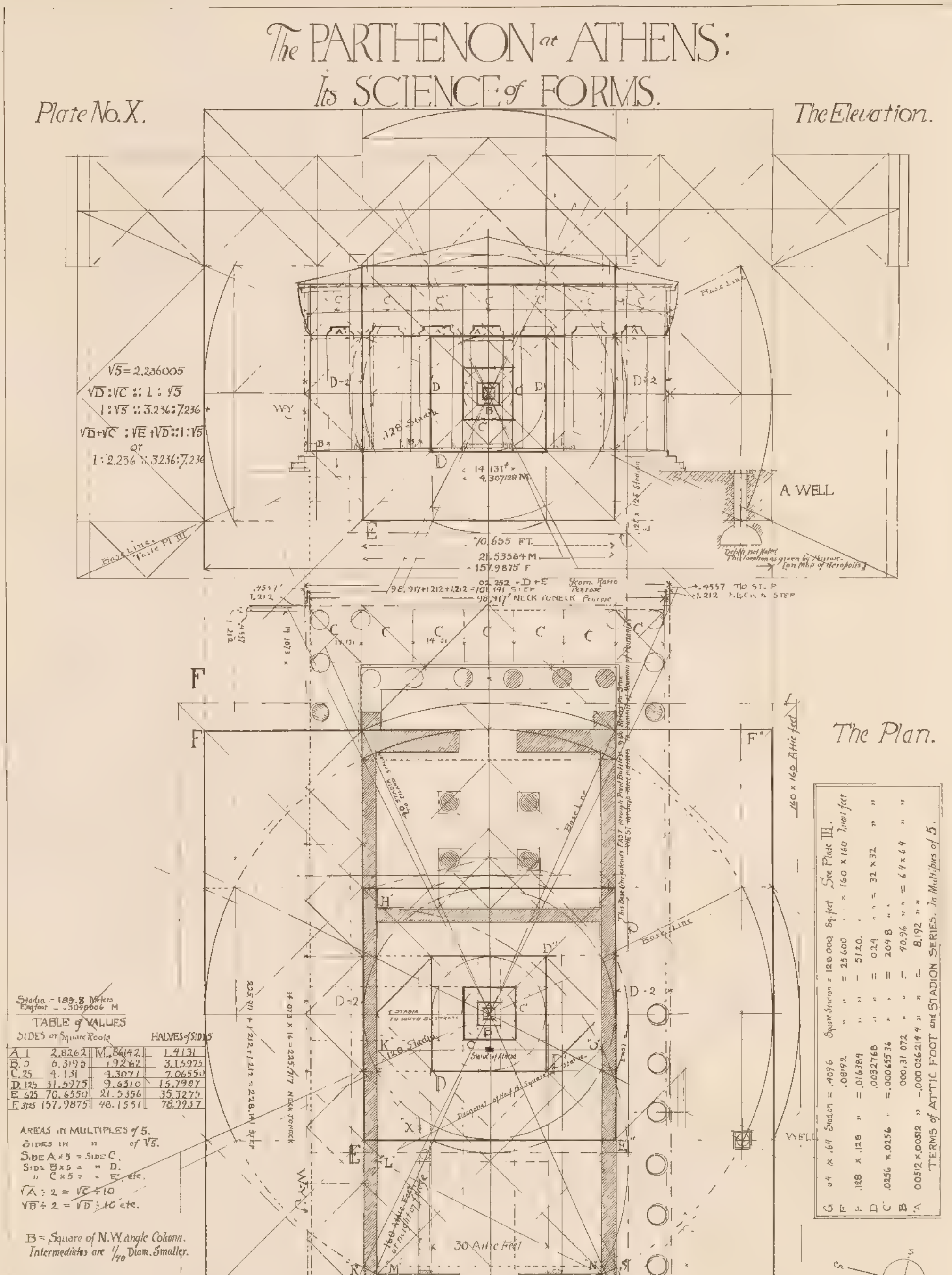
Step to Cincture

NOTE. Col. Height includes 0.2 Rise of step at axis.

Diagonal 32 Attic feet, is derived from Mean Height of Cinctures from datum.

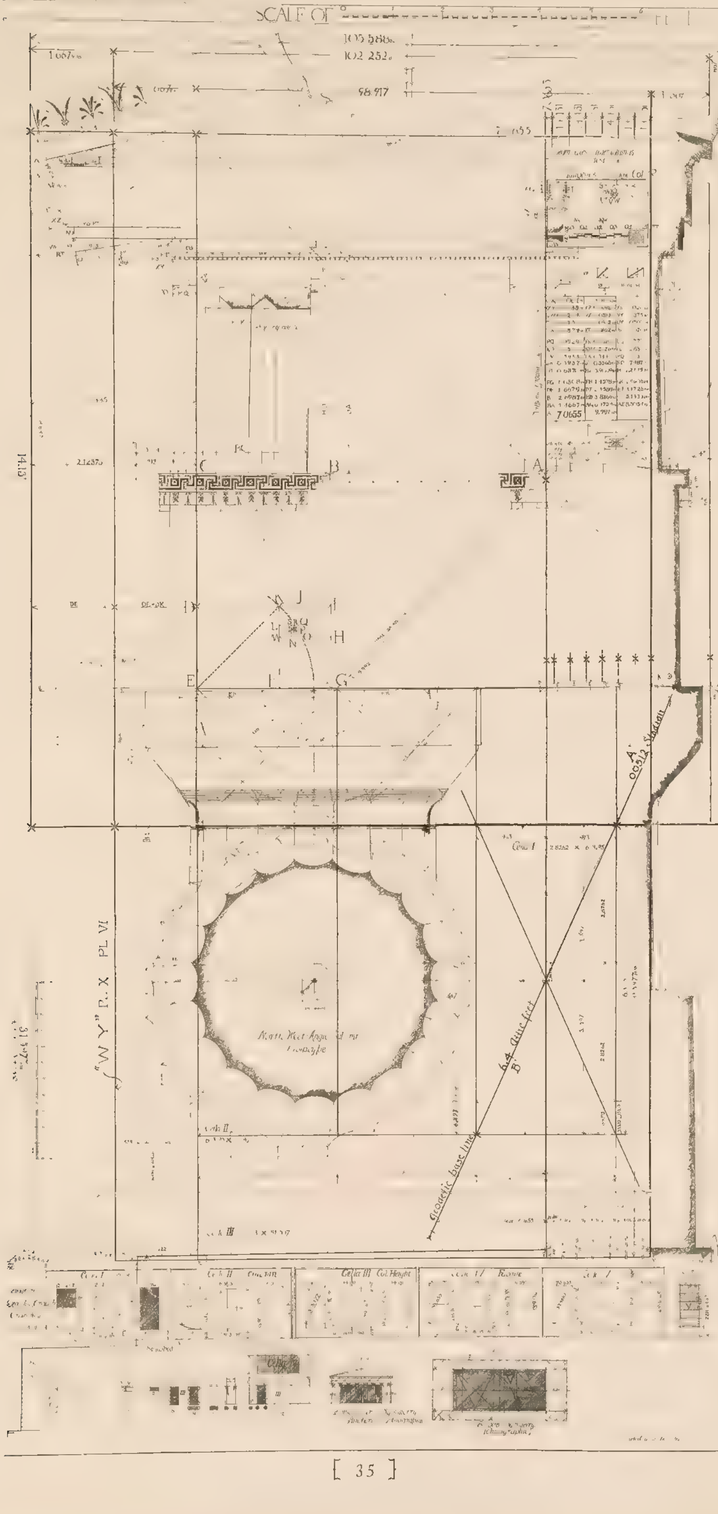
C" .0256 × .0256 Stadia = .00065536 Sq Stadia = 204.8 Sq. ft.
D" .0032768 " " = 1024. Sq. ft = 32 × 32 Attic Ft.

F.W.G. Del.


The PARTHENON at ATHENS:
Its SCIENCE of FORMS.
Plate No. X.
The Elevation.
√5 = 2.236005
√D : √C :: 1 : √5
1 : √5 :: 3.236 : 7.236
√D+√C : √E + √D :: 1 : √5
or
1 : 2.236 :: 3.236 : 7.236
128 Stadia
A WELL
70.655 FT.
21.53564 M.
157.9875 F
98.917 NECK TO NECK Penrose
The Plan.
160 x 160 Attic feet
Stadia - 189.8 Meters
Eng foot - .3048006 M
TABLE of VALUES
SIDES or Square Roots
HALVES of SIDES
A 1 2.8262 M. .86142 1.4131
B 5 6.3195 1.9262 3.15975
C 25 4.131 4.3071 7.06550
D 125 31.5975 9.6310 15.7987
E 625 70.6550 21.5356 35.3275
F 3125 157.9875 48.1551 78.9937
AREAS in MULTIPLES of 5.
SIDES in " of √5.
SIDE A x 5 = SIDE C.
SIDE B x 5 = " D.
" C x 5 = " E. etc.
√A : 2 = √C ÷ 10
√B ÷ 2 = √D ÷ 10 etc.
B = Square of N.W. angle Column.
Intermediates are 1/40 Diam. Smaller.
DETAIL
128 Stadia
30 Attic Feet
69.59
70.655 Penrose Feet
WELL
See Plate III.
TERMS of ATTIC FOOT and STADION SERIES. In Multiples of 5.
Note: The Geometric Relations are observed at the height of the Capitals, Not at Pavement level. They are attained by the rise of the Steps, The entasis and inclination of Columns and rise of the Entablature
R.W.G. Del.

THE TEMPLE AT ATHENS AND MECHANICAL LAW

PLATE XI



Of this book 225 copies were printed from Linotype slugs
at The Printing House of William Edwin Rudge
New York City